Astronomy & Astrophysics Challenge

September - December 2020

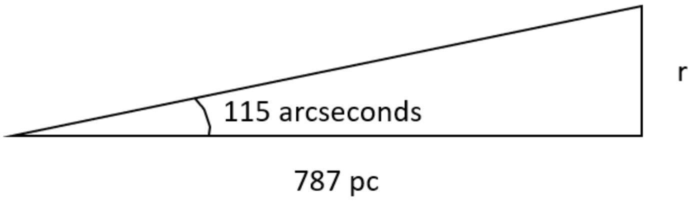
Solutions and marking guidelines

- The total mark for each question is in **bold** on the right-hand side of the table. The breakdown of the mark is below it.
- There is an explanation for each correct answer for the multiple-choice questions. However, the students are only required to write the letter corresponding to the right answer.
- In Section C, students should attempt **either** Qu 13 **or** Qu 14. If both are attempted, consider the question with the higher mark.
- Answers to two or three significant figures are generally acceptable. The solution may give more than that, especially for intermediate stages, to make the calculation clear.
- There are multiple ways to solve some of the questions; please accept all good solutions that arrive at the correct answer. Students getting the answer in a box will get all the marks available for that calculation / part of the question (students may not explicitly calculate the intermediate stages, and should not be penalised for this so long as their argument is clear)

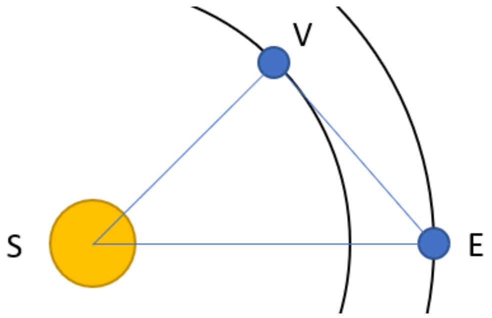
Question	Answer	Mark
Section A		**10**
1.	D Halley's comet was last in the inner solar system in 1986 and famously has an orbital period of 75.3 years so will next be visible to the naked eye in 2061.	1
2.	D Since Betelgeuse is a supergiant star, it pulsates and so varies in brightness anyway – this change in brightness was far more than usual, although few scientists really thought it might be about to go supernova. It is likely the change in brightness was due to a large ejection of superheated material from its surface that cooled into a dust cloud that blocked the light from about a quarter of it.	1
3.	B Beyond the core of a spiral galaxy, the rotational velocity is largely constant, giving a flat rotation curve – this can only be possible if there is considerable mass away from the centre of the galaxy, which is in contradiction with where the mass of stars appears to be concentrated, hence dark matter.	1

<table>
<tr><td>4.</td><td>A
Recognising you are given the angular diameter, then (working in degrees)

$$r = 787\tan\left(\frac{115}{3600}\right) = 0.44\text{ pc}$$



(Students may use the small angle approximation, so long as they remember to convert from degrees to radians.)</td><td>1</td></tr>
<tr><td>5.</td><td>A
Ophiuchus is on the ecliptic (and could be thought of as a bonus zodiacal constellation) whilst Aquila and Andromeda are both 15° - 30° above it.</td><td>1</td></tr>
<tr><td>6.</td><td>B
A star with a declination equal to the latitude of the observing location would culminate at the zenith (directly overhead), so Capella culminates highest as its declination is closest to 52°.</td><td>1</td></tr>
<tr><td>7.</td><td>A
We need to consider the situation when Venus is the greatest angular distance from the Sun to find out how far it can be from the Sun's constellation



Angle $S\hat{E}V$ is maximum when $S\hat{V}E = 90°$ so given $SV = 0.723$ au and $SE = 1$ au

$$\therefore S\hat{E}V = \sin^{-1}\left(\frac{0.723}{1}\right) = 46.3°$$

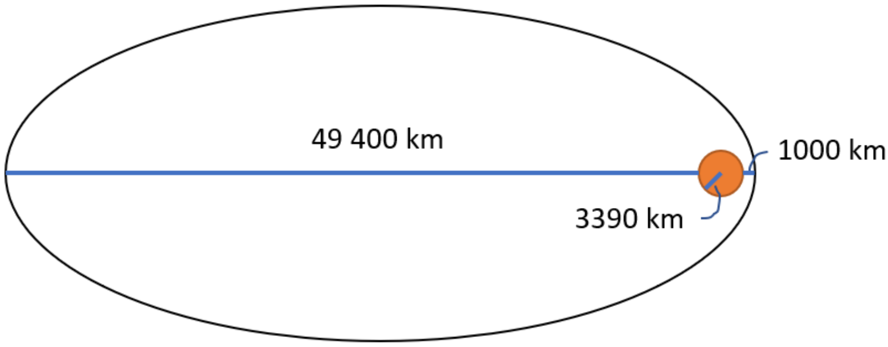
There are twelve zodiacal constellations, occupying about 30° each, so Venus can be no further than two constellations away from the Sun (i.e. < 60°). The twelve are (centred around Pisces):
Libra – Scorpio – Sagittarius – Capricorn – Aquarius – Pisces – Aries – Taurus – Gemini – Cancer – Leo – Virgo
Consequently, the only option within +/- 2 constellations of Pisces is Aries. (So in this situation it could be anywhere between Capricorn and Taurus.)</td><td>1</td></tr>
<tr><td>8.</td><td>B
A dimming of 0.0557 magnitudes means the new brightness is

$$b_{\text{new}} = 10^{-0.4(0.0557)}b_{\text{original}} = 0.950b_{\text{original}}$$

The size of the dip is proportional to the cross-sectional area of the star blocked out by the planet,

$$\therefore 1 - 0.950 = \frac{\pi R_P^2}{\pi R_S^2} \quad \therefore \frac{R_P}{R_S} = \sqrt{0.050} = 0.224$$</td><td>1</td></tr>
<tr><td>9.</td><td>C
Using the given formula:

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi r_S^3} = \frac{M}{\frac{4}{3}\pi\left(\frac{2GM}{c^2}\right)^3} = \frac{3c^6}{32\pi G^3 M^2}$$</td><td>1</td></tr>
</table>

	Putting in the numbers: $$\rho = \frac{3 \times (3.00 \times 10^8)^6}{32\pi \times (6.67 \times 10^{-11})^3 \times (4.15 \times 10^6 \times 1.99 \times 10^{30})^2}$$ $$= 1.07 \times 10^6 \text{ kg m}^{-3}$$	
10.	B  Semi-major axis, $a = \frac{1}{2}(49400 + 1000 + 2 \times 3390) = 28590$ km Using the formula given on page 2: $$T = \sqrt{\frac{4\pi^2}{GM}a^3} = \sqrt{\frac{4\pi^2}{6.67 \times 10^{-11} \times 6.39 \times 10^{23}} \times (28590 \times 10^3)^3}$$ $$= 1.47 \times 10^5 \text{ s} = 40.9 \text{ hours}$$	1
Section B		**10**
11.	a) The intensity of the light from the star on the planet is: $$b = \frac{L}{4\pi d^2} = \frac{0.0233 \times 3.85 \times 10^{26}}{4\pi \times (0.163 \times 1.50 \times 10^{11})^2} = 1.19 \times 10^3 \text{ W m}^{-2}$$ Multiplying by the cross-sectional area gives the total power incident: $$L_{\text{incident}} = b \times \pi R_P^2 = 1.19 \times 10^3 \times \pi \times (1.19 \times 6.37 \times 10^6)^2$$ $$= \boxed{2.16 \times 10^{17} \text{ W}}$$	**2** 1 1
	b) For thermal equilibrium, $L_{\text{incident}} = L_{\text{emitted}}$ $$\therefore L_{\text{incident}} = 4\pi R_P^2 \sigma T_P^4 \quad \therefore T_P = \sqrt[4]{\frac{L_{\text{incident}}}{4\pi R_P^2 \sigma}}$$ $$\therefore T_P = \sqrt[4]{\frac{2.16 \times 10^{17}}{4\pi \times (1.19 \times 6.37 \times 10^6)^2 \times 5.67 \times 10^{-8}}}$$ $$= 269.4 \text{ K} = \boxed{-3.6\ °\text{C}}$$ Lose 0.5 marks if they leave the answer in kelvin [Allow full ecf in this section for their answer to part a)]	**2** 1 1

	c) The exoplanet may well have an atmosphere keeping its surface temperature higher than predicted by our model **OR** higher atmospheric pressure, so water is still liquid at that temperature [Accept any valid idea as to why water could be liquid on its surface – refuse answers that would imply it is even colder e.g. does not absorb all incoming light]	**1** 1
12.	a) Recognising that they need to covert from Wh into J, $$t = \frac{10 \times 3600}{390} = \boxed{92\text{ s}}$$ This is very close to the planned maximum flight time of 90 s. About 20% of the flight will be in high power mode of around 510 W, during take-off and steering manoeuvres, whilst 80% will be travelling at a constant height in a straight line, with a power output of around 360 W.	**1** 1
	b) For objects in the Solar System, Kepler's 3rd Law is $T^2 = a^3$ if T is in years and a is in au, so $$a = \sqrt[3]{\left(\frac{687}{365}\right)^2} = 1.52\text{ au}$$ [Doing it this way gives 1.524 au, whilst using the full version of Kepler's 3rd Law in SI units gives 1.520 au – allow slight differences in the following numbers accordingly] Calculating the incident intensity at the surface of Mars: $$b = \frac{L_\odot}{4\pi a^2} = \frac{3.85 \times 10^{26}}{4\pi \times (1.524 \times 1.50 \times 10^{11})^2} = 586\text{ W m}^{-2}$$ Calculating the received power on the solar panel: $$P_{\text{received}} = b \times \text{area} \times \text{efficiency} = 586 \times \frac{544}{10^4} \times 0.3 = 9.56\text{ W}$$ Hence the charging time: $$t_{\text{charge}} = \frac{10\text{ Wh}}{P_{\text{received}}} = \frac{10 \times 3600}{9.56} = 3765\text{ s} = \boxed{62.7\text{ mins}}$$ [Must be in mins for the final mark]	**4** 1 1 1 1

Section C		**10**
13.	a) Recognising that they need to convert H_0 into SI units: $H_0 = 67.36\ \mathrm{km\ s^{-1}\ Mpc^{-1}} = 67.36 \times \frac{1000}{10^6 \times 3.09 \times 10^{16}} = 2.180 \times 10^{-18}\ \mathrm{s^{-1}}$ We can then put this into the given equation: $\rho_0 = \frac{3H_0^2}{8\pi G} = \frac{3 \times (2.180 \times 10^{-18})^2}{8\pi \times 6.67 \times 10^{-11}} = \boxed{8.50 \times 10^{-27}\ \mathrm{kg\ m^{-3}}}$ This corresponds to only about 5 hydrogen atoms per cubic metre!	**1** 0.5 0.5
	b) Since $\Omega \propto \rho$, we can look at the scaling relations for ρ and hence work out the scale factor first. At time t_{DE}: $\rho_\Lambda = \rho_m \therefore \Omega_\Lambda = \Omega_m$ $\therefore \Omega_{\Lambda,0} = \Omega_{m,0} a_{DE}^{-3}$ $\therefore a_{DE} = \sqrt[3]{\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}} = \sqrt[3]{\frac{0.3153}{0.6847}} = 0.7722$ Given we know how scale factor varies with time in a dark-energy-dominated epoch, as well as the current age of the Universe, we can now use a_{DE} to work out the time when this epoch started Converting t_0 into seconds, $t_0 = 13.80 \times 10^9 \times 365 \times 24 \times 60 \times 60 = 4.352 \times 10^{17}\ \mathrm{s}$ $\frac{a_{DE}}{a_0} = \frac{e^{H_0 t_{DE}}}{e^{H_0 t_0}}$ $t_{DE} = \frac{\ln(a_{DE} e^{H_0 t_0})}{H_0} = \frac{\ln\left(0.772 \times e^{2.18 \times 10^{-18} \times 4.35 \times 10^{17}}\right)}{2.18 \times 10^{-18}} = 3.17 \times 10^{17}\ \mathrm{s} = \boxed{10.0\ \mathrm{Gyr}}$ [Must be in Gyr for the final mark] This corresponds to a redshift of 0.295, and shows the transition happened about 1 Gyr after the Solar System was formed. We have used that the Hubble parameter is constant, as would be expected if $\Omega_\Lambda \approx 1$, although this would not be true throughout the period (especially as we approach t_{DE}), but this simple model gets close to the accepted value.	**4** 0.5 0.5 1 0.5 0.5 1

c) **5**

First, we can turn the given redshift into a scale factor:

$$a_{eq} = \left(1 + z_{eq}\right)^{-1} = (1 + 3402)^{-1} = 2.939 \times 10^{-4}$$ 0.5

Given the time and scale factor at the end of the matter-dominated epoch (t_{DE} and a_{DE}), the scale factor at the beginning of the epoch (a_{eq}), and the scaling relation for how the scale factor varies with time during the epoch ($a \propto t^{2/3}$), we can work out t_{eq}:

$$\frac{a_{eq}}{a_{DE}} = \frac{t_{eq}^{2/3}}{t_{DE}^{2/3}}$$ 0.5

$$\therefore t_{eq} = \left(\frac{a_{eq}}{a_{DE}} t_{DE}^{2/3}\right)^{3/2} = \left(\frac{2.939 \times 10^{-4}}{0.7722} \times (3.17 \times 10^{17})^{2/3}\right)^{3/2}$$

$$= \boxed{2.35 \times 10^{12}\ \text{s}} = 74.5 \times 10^{3}\ \text{years}$$ 1

[Accept any units for t_{eq} for this mark]

First, we can work out the density of dark energy today:

$$\rho_{\Lambda,0} = \Omega_{\Lambda,0}\rho_0 = 0.6847 \times 8.50 \times 10^{-27} = 5.82 \times 10^{-27}\ \text{kg m}^{-3}$$ 0.5

By definition, at t_{DE} then $\rho_{m,DE} = \rho_{\Lambda,DE} = \rho_{\Lambda,0} = 5.82 \times 10^{-27}\ \text{kg m}^{-3}$ 0.5

Given that for the matter-dominated epoch $\rho_m \propto a^{-3}$,

$$\frac{\rho_{m,eq}}{\rho_{m,DE}} = \left(\frac{a_{eq}}{a_{DE}}\right)^{-3}$$ 0.5

$$\therefore \rho_{m,eq} = \left(\frac{2.939 \times 10^{-4}}{0.7722}\right)^{-3} \times 5.82 \times 10^{-27} = 1.06 \times 10^{-16}\ \text{kg m}^{-3}$$ 0.5

By definition, at t_{eq} then matter only represents half the density of the Universe and so

$$\rho_{eq} = 2\rho_{m,eq} = \boxed{2.11 \times 10^{-16}\ \text{kg m}^{-3}}$$ 1

(Students may also combine $\rho_m \propto a^{-3}$ with $a \propto t^{2/3}$ to get $\rho_m \propto t^{-2}$ and hence set up a similar form of calculation to the penultimate mark. Allow full marks for this approach)

<table>
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<td>14.</td>
<td>a)

Given the eccentricity and the perihelion distance, we can use the formula from page 2 to work out the semi-major axis:

$$r_{peri} = a(1-e) \therefore a = \frac{r_{peri}}{1-e} = \frac{0.294649}{1-0.999188} = 362.9 \text{ au}$$
From this we can work out the period of the orbit using Kepler's 3rd Law. For objects in the Solar System, if T is in years and a is in au, then

$$T^2 = a^3 \therefore T = \sqrt{a^3} = \sqrt{362.9^3} = 6912.31 \text{ years}$$
(Using the full version in SI units gives $T = 6944.55$ years)

The perihelion on 3rd July corresponds to approximately halfway through the year 2020, so the starting date is $T_0 \approx 2020.5$ and thus the year of the next perihelion is

$$T_{\text{next}} = T_0 + T = 2020.5 + 6912.31 = 8932.81 = \boxed{8932}$$
(Using the other value of T gives $T_{\text{next}} = 8965.05 = \boxed{8965}$)

[Lose 0.5 marks if they do not take into account how far we are through 2020 already and hence get 8964]

In practice the orbital characteristics will get better known through observations, and since the eccentricity is so high that can shift the date of the next perihelion by several decades.</td>
<td>3

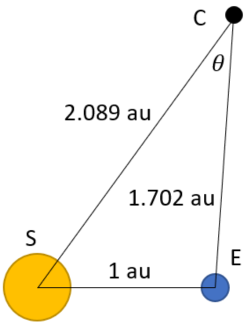
1

1

1</td>
</tr>
<tr>
<td></td>
<td>b)

Calculating the aphelion distance using the formula from page 2:

$$r_{aph} = a(1+e) = 362.9 \times (1 + 0.999188) = 725.4 \text{ au}$$
Calculating the phase angle on the discovery date using the cosine rule:



$$SE^2 = SC^2 + EC^2 - 2 \times SC \times EC \cos\theta$$
$$\therefore \theta = \cos^{-1}\left(\frac{SE^2 - SC^2 - EC^2}{-2 \times SC \times EC}\right)$$
$$\therefore \theta = \cos^{-1}\left(\frac{1^2 - 2.089^2 - 1.702^2}{-2 \times 2.089 \times 1.702}\right) = 28.3°$$
Evaluating $p(\theta)$ at both aphelion and in the discovery location:

$$p(\theta)_{aph} = p(0°) = B$$
$$p(\theta)_{disc} = p(28.3°) = 0.893B$$</td>
<td>7

1

1

0.5
0.5</td>
</tr>
</table>

	Assuming a cross-sectional area for the comet of A, we can calculate the total power incident for a given distance from the Sun, d_S:	
	$$P_{inc} = b_{inc} \times A = \frac{L_\odot}{4\pi d_S^2} \times A$$	0.5
	We can now use this to work out the brightness (i.e. intensity) of the light reflected from the comet as observed a given distance from the Earth, d_E:	
	$$b_{ref} = \frac{P_{ref}}{4\pi d_E^2} = \frac{\frac{L_\odot}{4\pi d_S^2} \times A \times p(\theta)}{4\pi d_E^2} = \frac{L_\odot A p(\theta)}{16\pi d_S^2 d_E^2}$$	0.5
	Considering the ratio of the reflected brightness for the two locations:	
	$$\frac{b_{aph}}{b_{disc}} = \frac{p(\theta)_{aph} d_{S,disc}^2 d_{E,disc}^2}{p(\theta)_{disc} d_{S,aph}^2 d_{E,aph}^2}$$	
	$$= \frac{B \times 2.089^2 \times 1.702^2}{0.893B \times 725.4^2 \times 724.4^2} = 5.13 \times 10^{-11}$$	1
	[Allow $d_{E,aph} = 726.4$ au too giving 5.10×10^{-11}, but penalise by 0.5 marks any other value]	
	We can now use this flux ratio to work out the magnitude at aphelion:	
	$$m_{aph} = m_{disc} - 2.5\log\left(\frac{b_{aph}}{b_{disc}}\right) = 18.0 - 2.5\log(5.13 \times 10^{-11}) = \boxed{43.7}$$	1
	This is much **fainter** than the Hubble Space Telescope's limiting magnitude so would **not** be visible	1